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# Quantum Tunneling in an Order-Parameter-Preserving Antiferromagnet

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**Abstract** We have studied quantum tunneling in an order-parameter-preserving antiferromagnet with the help of Holstein-Primakoff transformation. It is found that, when the system being prepared in a coherent state, there exist the quantum tunneling between lattices $k$ and $k+1$, $k$ and $k-1$, respectively. In particular, when the lattice is infinitely long and the spin excitations are in the long-wavelength limit, quantum tunneling disappear between lattices $k$ and $k+1$, and that $k$ and $k-1$, in this case the magnetic soliton appears.

**Keywords** Order-parameter-preserving antiferromagnet · Quantum tunneling

## 1 Introduction

Mikeska et al have reviewed the solitary excitations in one-dimensional magnets [1], Liu et al. have studied solitons in an order-parameter-preserving antiferromagnet (OPP-AFM) by introducing the Dyson-Maleev transformation [2]. In the present paper, we shall study quantum tunneling in an order-parameter-preserving antiferromagnet with the help of Holstein-Primakoff transformation.

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## 2 Hamiltonian in an Order-Parameter-Preserving Antiferromagnet

The Hamiltonian describing an OPP-AFM in the external magnetic field is given by [3]

$$H = \sum_{\langle ij \rangle} \left[ J_{ij} S_i^z S_j^z - \frac{1}{2} \Delta_{ij} (S_i^+ S_j^+ + S_i^- S_j^-) \right] - h \sum_i S_i^2, \tag{1}$$

where $J_{ij}$ and $\Delta_{ij}$ are the exchange integrals for spins located at sites $i$, $j$ of the lattice and $h$ is the magnitude of the external magnetic field.

With the help of Holstein-Primakoff transformation for the low-lying spin excitations at low temperature: $S_i^+ \simeq \sqrt{2S}(1 - \frac{1}{4S} a_i^\dagger a_i + \cdots) a_i$, $S_i^- \simeq \sqrt{2S} a_i^\dagger (1 - \frac{1}{4S} a_i^\dagger a_i + \cdots)$, $S_i^z = (S - a_i^\dagger a_i)$, where $n_i = a_i^\dagger a_i = S - S_i^z$ is the local spin deviation operator and the boson operators $a_i$ satisfy the usual commutation relation: $[a_i, a_j^\dagger] = \delta_{ij}$, $[a_i, a_j] = [a_i^\dagger, a_j^\dagger] = 0$. The Hamiltonian (1) then reduces to

$$\begin{aligned} H = {} & -hNS(S+1) + \sum_{\langle ij \rangle} J_{ij} (S^2 - S a_i^\dagger a_i - S a_j^\dagger a_j + a_i^\dagger a_i a_j^\dagger a_j) - S \sum_{\langle ij \rangle} \Delta_{ij} (a_i a_j + a_i^\dagger a_j^\dagger) \\ & + \frac{1}{4} \sum_{\langle ij \rangle} \Delta_{ij} (a_i a_j^\dagger a_j^2 + a_i^\dagger a_i^2 a_j + a_i^\dagger a_j^{\dagger 2} a_j + a_i^{\dagger 2} a_i a_j^\dagger), \end{aligned} \tag{2}$$

where the higher order terms are neglected.

## 3 Quantum Tunneling in an Order-Parameter-Preserving Antiferromagnet

In this section, we shall study the quantum tunneling in an order-parameter-preserving antiferromagnet. The nonlinear interactions in Hamiltonian (2) generate the nonlinear magnetic excitations such as mixing of spin waves or magnetic solitons [4] depending on the initial setup of the state of the spin system. Here we are interested in the study of quantum tunneling induced by the nonlinear interactions. The ideal case is that the spin system in an order-parameter-preserving antiferromagnet should initially be prepared in a coherent state $|\{\psi_k\}\rangle = \Pi_k D(\alpha_k)|0\rangle$, where $D(\alpha_k) = \exp[\alpha_k a^\dagger - \alpha_k^* a]$, and the vacuum state $|0\rangle$ is the ground state.

Under the spin coherent state and using the time-dependent variation principle, the nonlinear motion equation of atomic number $\psi_k = \langle \psi | a_k^\dagger a_k | \psi \rangle$ on the lattice $k$ can be derived as

$$\begin{aligned} i\hbar \frac{\partial \psi_k}{\partial t} = {} & -2SJ\psi_k + 2J\psi_k (|\psi_{k+1}|^2 + |\psi_{k-1}|^2) \\ & - 2S\Delta(\psi_{k+1}^* + \psi_{k-1}^*) + \frac{\Delta}{2}(\psi_{k+1} + \psi_{k-1})\psi_k^2 \\ & + \frac{\Delta}{2}[|\psi_{k+1}|^2 \psi_{k+1}^* + |\psi_{k-1}|^2 \psi_{k-1}^* + 2|\psi_k|^2 (\psi_{k+1}^* + \psi_{k-1}^*)], \end{aligned} \tag{3}$$

where we have considered the system described by the Hamiltonian (1) containing only nearest-neighbour interactions and with $J_{ij} = J$, $\Delta_{ij} = \Delta$ for all spin pairs.

Similarly, we can obtain the motion equation of atomic number $\mu_{k+1} = \langle\psi|a_{k+1}^{\dagger}a_{k+1}|\psi\rangle$ and $\nu_{k-1} = \langle\psi|a_{k-1}^{\dagger}a_{k-1}|\psi\rangle$ on the lattice $k+1$ and $k-1$, respectively:

$$
\begin{aligned}
i\hbar\frac{\partial\mu_{k+1}}{\partial t} &= -2SJ\mu_{k+1} + 2J\mu_{k+1}(|\mu_{k+2}|^2 + |\mu_k|^2) \\
&\quad - 2S\Delta(\mu_{k+2}^* + \mu_k^*) + \frac{\Delta}{2}(\mu_{k+2} + \mu_k)\mu_{k+1}^2 \\
&\quad + \frac{\Delta}{2}[|\mu_{k+2}|^2\mu_{k+2}^* + |\mu_k|^2\mu_k^* + 2|\mu_{k+1}|^2(\mu_{k+2}^* + \mu_k^*)],
\end{aligned}
\tag{4}
$$

and

$$
\begin{aligned}
i\hbar\frac{\partial\nu_{k-1}}{\partial t} &= -2SJ\nu_{k-1} + 2J\nu_{k-1}(|\nu_k|^2 + |\nu_{k-2}|^2) - 2S\Delta(\nu_k^* + \nu_{k-2}^*) + \frac{\Delta}{2}(\nu_k + \nu_{k-2})\nu_{k-1}^2 \\
&\quad + \frac{\Delta}{2}[|\nu_k|^2\nu_k^* + |\nu_{k-2}|^2\nu_{k-2}^* + 2|\nu_{k-1}|^2(\nu_k^* + \nu_{k-2}^*)].
\end{aligned}
\tag{5}
$$

According to (3–5), we see that $\frac{\partial}{\partial t}(\psi_k - \mu_{k+1}) \neq \frac{\partial}{\partial t}(\psi_k - \nu_{k-1})$, which means that the tunnelings between lattices $k$ and $k+1$, $k$ and $k-1$ are general different. In particular, when the optical lattice is infinitely long and the spin excitations are in the long-wavelength limit, one has $\psi_k = \psi_{k+1} = \psi_{k-1}$, $\mu_k = \mu_{k+1} = \mu_{k-1}$, and $\nu_k = \nu_{k+1} = \nu_{k-1}$ in the continuum limit approximation, this shows that there does not exist the tunneling effect between lattices $k$ and $k+1$, and that $k$ and $k-1$. Correspondingly, the magnetic soliton appears.

## 4 Conclusions

In summary, we have studied quantum tunneling in an order-parameter-preserving antiferromagnet with the help of Holstein-Primakoff transformation. It is found that, when the system being prepared in a coherent state, there exist the quantum tunneling between lattices $k$ and $k+1$, $k$ and $k-1$, respectively. In particular, when the lattice is infinitely long and the spin excitations are in the long-wavelength limit, the quantum tunneling disappears between lattices $k$ and $k+1$, and that $k$ and $k-1$, in this case the magnetic soliton appears.

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